ON GENERAL EXPRESSIONS OF TEMPORAL MEANS OF MASS FLUX, SHEAR STRESS AND HEAT FLUX DUE TO THERMOACOUSTIC WAVES

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Introduction

Performance of thermoacoustic heat engines is determined crucially by various nonlinear phenomena. Among them, steady streaming of mass and energy impacts on it significantly. To aim at making its quantitative description, nonlinear theory of thermoacoustic waves in a channel and a tube has been developed by the author and a nonlinear diffusion-wave equation has been derived for a case that a diffusion layer is thick enough compared with a span length [1]. Within the same framework, this paper derives general expressions of temporal means of mass flux, shear stress and heat flux on a wall by assuming time-periodic wave. It is shown that the mean pressure is set down and that all mean values are expressed in terms of the mean pressure and its spatial gradients.

Nonlinear diffusion-wave equation and general expressions for the temporal mean values

When a typical diffusion layer is thick enough compared with a span length of a channel and a tube as in a gas in regenerators, propagation of nonlinear thermoacoustic waves therein is governed by the following diffusion-wave equation in an axial coordinate x and a time t as

$$\frac{\partial p'}{\partial t}-\frac{\partial}{\partial x}\left(\alpha_e\frac{\partial p'}{\partial x}\right)+\frac{\alpha_e}{T_e}\frac{\mathrm{d}T_e}{\mathrm{d}x}\frac{\partial p'}{\partial x}$$
$$+\left[n\gamma-(\gamma-1)\mathrm{Pr}\right]\frac{\alpha_e}{a_e^2}\frac{\partial^2 p'}{\partial t^2}-m(1+\beta+\mathrm{Pr})\frac{\alpha_e H^2}{\nu_e T_e}\frac{\mathrm{d}T_e}{\mathrm{d}x}\frac{\partial^2 p'}{\partial t\partial x}-\frac{p'}{p_0}\frac{\partial p'}{\partial t}-\frac{\alpha_e}{p_0}\left(\frac{\partial p'}{\partial x}\right)^2=0, \tag{1}$$

where p_0 and p' denote, respectively, a uniform pressure in a quiescent state and an excess pressure from it; T_e denotes a wall temperature non-uniform axially, which is assumed to be fixed temporarily; $\alpha_e(=ma_e^2H^2/n\gamma\nu_e)$ denotes a diffusivity of acoustic waves in a channel of span length of $2H$ and in a tube of radius $R(=H)$, respectively, with the pair of constants $m=2/5$ and $n=6/5$, and the one of $m=1/6$ and $n=8/6$, a_e, ν_e, β, γ and Pr being, respectively, an adiabatic sound speed, a kinematic viscosity, a factor in the temperature dependence of viscosity, the ratio of specific heats and Prandtl number.

Assuming time-periodic solutions to (1), it is found that

$$\frac{\alpha_e}{T_e}\widetilde{\left(\frac{\partial p'}{\partial x}\right)^2}=-p_0\frac{\partial}{\partial x}\left(\frac{\alpha_e}{T_e}\frac{\partial\widetilde{p'}}{\partial x}\right), \tag{2}$$

where a tilde designates a temporal mean over one period. This shows that the gradient of the mean pressure $\widetilde{p'}$ is of quadratic order in disturbance and that the mean of the pressure gradient squared is expressed in terms of the derivative of the mean pressure gradient. It is noted that finite effects of span length, represented by the first two terms on the second line in (1), do not contribute to (2).

In a similar fashion, mean values of the mass flux, shear stress and heat flux on the wall are expressed in terms of the mean of the products of spatial and/or temporal gradients of the excess pressure, which are in turn expressed in terms of the mean pressure gradient through (2). Hence once the mean excess pressure is available, all means are obtained in terms of it.

Summary

Because thermoacoustic heat engines do not consist of the regenerator only, (1) should be solved in conjunction with other nonlinear wave equations for propagation in a buffer tube, a heat exchanger and a tube without temperature gradient. In fact, the linearized version of (1) has been solved in this way to derive marginal conditions for the onset of thermoacoustic oscillations in a gas-filled tube [2]. Although no attempt to include the nonlinear terms has been made, it is expected to obtain autonomous excitation of thermoacoustic waves of finite amplitude. When these time-periodic waves are available, it is straightforward to have mean values due to steady streaming of mass and energy. Their expressions may illuminate a way how to control the streaming.

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